ADAPTIVE CONTROL OF FORMATION FLYING SPACECRAFT FOR INTERFEROMETRY

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Abstract: This paper presents an adaptive control system for coordination and control of a fleet of micro-spacecraft moving in formation. A fleet of spacecraft are given as a collection of systems which interact with each other in a cooperative manner to achieve a common objective. To provide a desired formation, basic mathematical models for controlled movement of rigid bodies in free space is presented followed by the adaptive control law for formation manipulations and formation keeping. The formation control performance in the presence of constant but unknown disturbances is illustrated by simulations. *Copyright ©1998 IFAC*

Keywords: Formation Flying, Separated Spacecraft, Interferometry Adaptive Control

1. INTRODUCTION

The concept of multiple coordinated spacecraft is emerging as an attractive alternative to the traditional single large spacecraft for both commercial satellite communications and space science missions. There are several advantages to this concept. The most important of these, is the ability to make the mission much more robust by eliminating single point failures. The multiple spacecraft approach will also impose less requirements and limitations on launch vehicles and thereby reducing the mission cost.

In the case of monolithic space interferometers, stringent requirements on control of a highly complex structures is a precursor to instruments ability for successful astrometry and imaging. In recent years, growing emphasis is placed on the concept of separated spacecraft interferometry (SSI). The SSI concept envisioned the collecting apertures to be located on separate spacecraft while central combining instruments to be located on yet another spacecraft. A virtual structure is therefore developed without the real need for maintaining the necessary structural rigidity. The SSI provides measurements unachievable with other techniques and allows long baseline lengths and orientation changes without a need for high precision control. However, separated spacecraft systems introduce new complexities and challenges. For example, initialization and stabilization of the formation are two new problems that have no analogue in the single spacecraft paradigm. Once these are achieved, the challenge of conducting a variety of precision rotations, formation contractions and expansions will be required to execute a science scenario. These maneuvers are required, for example, for interferometric imaging to fill in the plane of the sparse aperture formation. In addition to those maneuvers that are science driven, there must also be maneuvers for orbit maintenance, control strategies for cooperative attitude and station keeping, maneuvers for formation alignment and calibration, and the optimization of formation resources and perfor-

[1] This work was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

mance. Under NASA's New Millennium Program, the Deep Space Mission 3 mission (DS3) is a concept for a separated-spacecraft optical interferometer. The interferometer instrument will be distributed over three small spacecraft: two spacecraft will serve as collectors, directing starlight toward a third spacecraft which will combine the light and perform the interferometric detection. The mission is nominally deployed in a low-disturbance solar orbit to minimize the station-keeping burden. The interferometer baselines may be variable from 5 m to 1 km. Low-bandwidth corrections for 1 centimeter and 1 arc-minute station-keeping and relative attitude errors may be accomplished by spacecraft control with electric-propulsion or a cold-gas system; high-bandwidth corrections for 1 nanometer and 0.1 arc-second stationkeeping and relative attitude errors would be accomplished by feedforward to interferometer compensators and actuators in the combiner spacecraft.

Although control of a single spacecraft is based on well established control theory concepts and methodologies, the control systems for multiple spacecraft moving in formation require architectures which differ from those of conventional single spacecraft control systems. To provide a desired formation, basic mathematical models for controlled movement of each micro-spacecraft in free space is needed. The control laws for the coordination of spacecraft attitude during motion to achieve a specified objective (e.g. orient each spacecraft along a given direction) is an integral part of control laws for formation keeping and relative attitude alignment. This paper presents an adaptive control system for the coordination and control of a fleet of micro-spacecraft moving in formation in the presence of constant disturbances. A fleet of spacecraft are given as a collection of systems which interact with each other in a cooperative manner to achieve a common objective. The time-domain behavior of the feedback-controlled formation flying for typical low-Earth orbits is studied both analytically and via computer simulation. Emphasis is placed on the conditions for ensuring formation stability and control performance in the presence of various types of constant but unknown disturbances.

2. DYNAMICS OF MULTIPLE SPACECRAFT

Following the convention in [4], we regard each spacecraft as a point mass moving in free space under the influence of its environment, including the gravitational field, aerodynamics, solar radiation, as well as magnetic field [5]. We assume the fleet consists of N spacecraft. We shall use the following coordinate systems to derive the equations of motion for spacecraft in three-dimensional Euclidean space R^3: an inertial coordinate system $\mathcal{F}_0$ and a set of moving coordinate systems $\mathcal{F}_i$, $i = 1, \cdots, N$. The origin O_i and the axes of $\mathcal{F}_i$ are at the center of mass and along the principal axes of inertia of the i-th spacecraft respectively.

Let a be an arbitrary vector in a moving coordinate system, which rotates in $\mathcal{F}_0$ with angular velocity ω, the time derivative $\frac{da}{dt}$ of a with respect to $\mathcal{F}_0$ is related to the time derivative $\frac{da}{dt_i}$ of a with respect to $\mathcal{F}_i$ as follows:

$$\frac{da}{dt} = \frac{da}{dt_i} + \omega \times a. \tag{1}$$

First, we consider the translational motion of the spacecraft. In the coordination of spacecraft flying in formation, we are interested in the relative motion between a pair of spacecraft, say i and j. Suppose the positions of the mass centers of two spacecraft in $\mathcal{F}_0$ are r_i and r_j, respectively. Then their relative position is:

$$\rho_{ij} = r_j - r_i. \tag{2}$$

From Newton's law, the equation of motion for the j-th spacecraft relative to $\mathcal{F}_i$ is as follows:

$$\frac{dp_j}{dt} = f_{dj} + f_{cj} \tag{3}$$

where f_{dj} and f_{cj} are disturbance and control forces respectively; p_j is the linear momentum of the j-th spacecraft, defined as follows:

$$p_j := M_j v_j = M_j \frac{dr_j}{dt} = M_j \frac{d(r_i + \rho_{ij})}{dt} = M_j(v_i + \frac{d\rho_{ij}}{dt}), \tag{4}$$

M_j is the mass of the spacecraft j; v_j is its linear velocity in $\mathcal{F}_0$; r_i is also the origin of $\mathcal{F}_i$ in $\mathcal{F}_0$. From (3) and (4), one has

$$\frac{dv_i}{dt} + \frac{d^2\rho_{ij}}{dt^2} = \frac{f_{dj} + f_{cj}}{M_j} =: d_j + u_j, \tag{5}$$

where d_j and u_j are linear accelerations due to disturbance and control forces respectively:

$$d_j := \frac{f_{dj}}{M_j}, \quad u_j := \frac{f_{cj}}{M_j}. \tag{6}$$

In (5), let $j = i$. When $\rho_{ij} = 0$, we have

$$\frac{dv_i}{dt} = \frac{dv_i}{dt_i} + \omega_i \times v_i = d_i + u_i. \tag{7}$$

Therefore, the last two equations imply

$$\frac{d^2\rho_{ij}}{dt^2} = d_j + u_j - d_i - u_i. \tag{8}$$

Now using (1), we obtain

$$\frac{d\rho_{ij}}{dt} = \frac{d\rho_{ij}}{dt_i} + \omega_i \times \rho_{ij}, \qquad (9)$$

and

$$\frac{d^2\rho_{ij}}{dt^2} = \frac{d^2\rho_{ij}}{dt_i^2} + \frac{d\omega_i}{dt_i} \times \rho_{ij} + 2\omega_i \times \frac{d\rho_{ij}}{dt_i} + \omega_i \times (\omega_i \times \rho_{ij}). \qquad (10)$$

On the other hand, the dynamics of angular velocities of the spacecraft relative to $\mathcal{F}_0$ are described by the following Euler's equations:

$$\frac{d(I_i\omega_i)}{dt} = I_i \frac{d\omega_i}{dt_i} + \omega_i \times (I_i\omega_i) = \tau_i, \quad (11)$$

where $i = 1, \cdots, N$; I_i is the tensor of inertia associated with the i-th spacecraft; and

$$\tau_i := \tau_{di} + \tau_{ci} \qquad (12)$$

is the external torque applied to the spacecraft, including disturbance torque τ_{di} and control torque τ_{ci}. Equation (11) can be written as

$$\frac{d\omega_i}{dt_i} = I_i^{-1}(\tau_i - \omega_i \times (I_i\omega_i)). \qquad (13)$$

Using the above relation, (10) and (8) imply

$$\frac{d^2\rho_{ij}}{dt_i^2} + I_i^{-1}(\tau_i - \omega_i \times (I_i\omega_i)) \times \rho_{ij} + 2\omega_i \times \frac{d\rho_{ij}}{dt_i} + \omega_i \times (\omega_i \times \rho_{ij}) = d_j + u_j - d_i - u_i. (14)$$

Therefore, (13) and (14) completely describe the dynamics of the relations with each other among the N spacecraft.

3. FORMATION KEEPING PROBLEM

A fleet of spacecraft flying in formation can be achieved in many ways, and various schemes for generating certain formation patterns are discussed in [4]. As control law design for formation-keeping of each formation pattern is similar, in what follows, we consider a particular formation pattern.

3.1 *Dynamic Equations for Formation Keeping*

Suppose several spacecraft are chosen as leaders for formation flying. There motions serve as a skeletal pattern for the fleet. The desired motion for the remaining spacecraft may be determined by the spacecraft in their specified neighborhood. The following is one of the methods for specifying the motion of a spacecraft [4].

Let $\mathcal{I}_i(t) \subset \{1, 2, \cdots, N\} \backslash \{i\}$ denote the index set consisting of the labels of specified N_i neighbors of the i-th spacecraft, whose position at time t is $r_i(t)$. Let $r_j(t)$ is the position of the j-th spacecraft at time t for $j \in \mathcal{I}_i(t)$. Let $\rho_{ij}(t) = r_j(t) - r_i(t)$. We may set the desired motion $d_i(t)$ for the i-th spacecraft as follows:

$$d_i(t) = \frac{1}{N_i} \sum_{j \in \mathcal{I}_i(t)} r_j(t). \qquad (15)$$

In formation-keeping control design, the objective calls for the i-th spacecraft to track the motion $d_i(t)$ defined by (15). The tracking error for the i-th spacecraft is defined by

$$E_i(t) = d_i(t) - r_i(t) = \frac{1}{N_i} \sum_{j \in \mathcal{I}_i(t)} r_j(t) - r_i(t) = \frac{1}{N_i} \sum_{j \in \mathcal{I}_i(t)} (r_j(t) - r_i(t)) = \frac{1}{N_i} \sum_{j \in \mathcal{I}_i(t)} \rho_{ij}(t) (16)$$

Note that each $\rho_{ij}(t)$ satisfies (14). To facilitate the control design, the dynamics for the tracking error $E_i(t)$ defined in (16) is derived using (14) as follows:

$$\frac{d^2E_i}{dt_i^2} + I_i^{-1}(\tau_i - \omega_i \times (I_i\omega_i)) \times E_i + 2\omega_i \times \frac{dE_j}{dt_i} + \omega_i \times (\omega_i \times E_i) = \frac{1}{N_i} \sum_{j \in \mathcal{I}_i(t)} (d_j - d_i + u_j) - u_i. \qquad (17)$$

The formation keeping problem can be defined as follows,

Definition 1. Formation Keeping Consider a fleet of N spacecraft, the reference position of the i-th spacecraft is defined as $d_i(t)$ in (15). The formation keeping is to design a feedback controller such that the actual position $r_i(t)$ for the spacecraft closely tracks $d_i(t)$ in the sense that the tracking error $E_i(t) = d_i(t) - r_i(t)$ satisfies

$$\lim_{t \to \infty} E_i(t) \to 0. \qquad (18)$$

3.2 *Different Sources of Disturbances*

The disturbances entering the system equations can be classified as disturbance torques and disturbance forces. Both are due to the gravitational field, solar radiation, aerodynamics (for low orbit mission), or magnetic field. The external torque τ_i consists of disturbance torque τ_{di} and control torque τ_{ci}: $\tau_i = \tau_{di} + \tau_{ci}$ The disturbance torque

τ_{di} includes gravity gradient, solar radiation, aerodynamic, and magnetic torques (for a detailed discussion, see [5]). Those torques vary slowly during flight. Therefore, τ_{di} can be assumed to be a constant parameter vector.

The disturbance accelerations d_j $(j \in \mathcal{I}_i(t))$ and d_i include those generated by gravity gradient, solar radiation, aerodynamic, and magnetic forces. Their variations can also be neglected. We define

$$d = \frac{1}{N_i} \sum_{j \in \mathcal{I}_i(t)} (d_j - d_i), \tag{19}$$

then it can be viewed as a constant parameter vector. With above discussion, in the following design, we define

$$\theta := \begin{bmatrix} \tau_{di} \\ d \end{bmatrix} \tag{20}$$

as a constant parameter vector.

3.3 *State-space Equation and Feedback Linearization*

To facilitate control law design, we first give the state-space equation of the relevant dynamics for the formation keeping problem. Let the state vector be defined by $x_1 = E_i$, $x_2 = \dot{E}_i := \frac{dE_i}{dt_i}$, $x_3 = \omega_i$, and the output vector be $y = E_i$.

From (17) and (13), we have the following 9-th order state space equation for the formation keeping problem:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_0(x_1, x_2, x_3, u) + f_1(x_1)\theta - u_i \\ \dot{x}_3 = \xi(x_3, \tau_i) \\ y = x_1 \end{cases} \tag{21}$$

where $f_0(x_1, x_2, x_3) := I_i^{-1}(x_3 \times (I_i x_3) - \tau_{ci}) \times x_1 - 2x_3 \times x_2 - x_3 \times (x_3 \times x_1) + u$, and $u := \frac{1}{N_i}\sum_{j \in \mathcal{I}_i(t)} u_j$, which is known in this design;

$$f_1(x_1) := \left[-I_i^{-1} M(x_1)\right] I \tag{22}$$

with $M(x_1)$ being a 3×3-matrix function such that

$$x_1 \times v = M(x_1) v \tag{23}$$

for all $v \in \Re^3$; and

$$\xi(x_3, \tau_i) := I_i^{-1}(\tau_i - x_3 \times (I_i x_3)). \tag{24}$$

Note that if θ is measurable, then the system (21) is input-output linearizable. In fact, if we replace u_i in (21)

$$u_i = f_0(x_1, x_2, x_3) - f_1(x_1)\theta - v \tag{25}$$

,then the system is input-output linearized, and the input-output dynamics with state $x = [x_1 x_2 x_3]$ is

$$\begin{cases} \dot{x} = Ax + Bv \\ y = Cx \end{cases} \tag{26}$$

where

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad C = \begin{bmatrix} I & 0 \end{bmatrix}, \tag{27}$$

and the zero dynamics is

$$\dot{x}_3 = \xi(x_3, \tau_i) \tag{28}$$

The system can be designed using linear techniques, one of the approaches is to use Lyapunov technique which was presented in [4].

4. ADAPTIVE CONTROL APPROACH

Actually, in the control design for formation keeping, we don't need to directly measure the unknown parameter θ. This can be achieved using adaptive control method.

4.1 *Problem Formulation*

Here, we formulate the formation keeping problem without the measurement of the disturbances as an adaptive control problem. In the design, we assume that the information E_i, $\dot{E}_i$ and ω_i are available. If the parameter θ is known, we can use the foregoing input-output linearization result. Suppose that the following feedback control is chosen

$$v = Fx = F_1 x_1 + F_2 x_2 \tag{29}$$

so that the closed-loop system $\dot{x} = (A + BF)x$ is asymptotically stable, then there exists a Lyapunov function

$$V(x) = x^T P x \tag{30}$$

such that the following Lyapunov equation holds,

$$P(A + BF) + (A + BF)^T P = -Q < 0. \tag{31}$$

Moreover,

$$\lim_{t \to \infty} E_i(t) = \lim_{t \to \infty} x_1(t) = 0. \tag{32}$$

The resulting feedback controller is

$$u_i = f_0(x_1, x_2, x_3) + f_1(x_1)\theta + F_1 x_1 + F_2 x_2 \tag{33}$$

Since the parameter θ is not known in this design, control law (33) cannot be implemented. But we

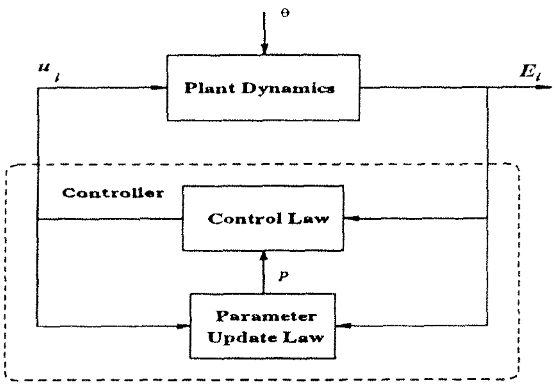

Fig. 1. Adaptive Control Law for the i-th Spacecraft

can still use the controller with the parameters replaced by their estimates p. Let the parameter update law be given by:

$$\dot{p} = \eta(x, u), \tag{34}$$

where p is the parameter estimate. The control input becomes

$$u_i = f_0(x_1, x_2, x_3) + f_1(x_1)p + F_1 x_1 + F_2 x_2 \tag{35}$$

The block diagram for the adaptive system is illustrated in Fig. 1.

The adaptive control for the formation keeping problem is defined as follows:

Definition 2. Adaptive Control Design update law (34) such that control law (35) insures $E_i(t) \to 0$ as $t \to \infty$ for all unknown constant disturbance vector θ.

4.2 *Derivation of Adaptive Control Laws*

Now, we shall derive an update law (34) for the parameter estimation using Lyapunov technique. We first apply control law (35) with estimated parameter p. The resulting system becomes

$$\ddot{x}_1 + F_2 \dot{x}_1 + F_1 x_1 = -f_1(x_1)(p - \theta). \tag{36}$$

To derive an update law such that $x_1(t) \to 0$ as $t \to \infty$, take a Lyapunov function as follows:

$$V(x, p) = x^T P x + \|p - \theta\|^2. \tag{37}$$

Then

$$\begin{aligned}\dot{V}(x,p) &= 2x^T P((A + BF)x + \\ &\begin{bmatrix} 0 \\ f_1(x_1)(p-\theta) \end{bmatrix} + 2(p-\theta)^T \dot{p} = \\ &x^T(P(A+BF) + (A+BF)^T P)x + \\ &2(p-\theta)^T(\dot{p} - \begin{bmatrix} 0 & f_1^T(x_1) \end{bmatrix} Px)\end{aligned} \tag{38}$$

Now, let

$$\dot{p} = \begin{bmatrix} 0 f_1^T(x_1) \end{bmatrix} Px, \tag{39}$$

then

$$\begin{aligned}\dot{V}(x,p) = x^T(P(A+BF) + (A+BF)^T P)x = \\ -x^T Q x \le 0.\end{aligned} \tag{40}$$

Next, we show that the adaptive control law (34)-(35):

$$\begin{cases} \dot{p} = \begin{bmatrix} 0 & f_1^T(x_1) \end{bmatrix} Px \\ u_i = f_0(x_1, x_2, x_3) + f_1(x_1)p + F_1 x_1 + F_2 x_2, \end{cases} \tag{41}$$

indeed solves the adaptive control problem for formation keeping. First we show that $x(t) \to 0$ as $t \to \infty$ with the implementation of the above control law. Note from (40) that the Lyapunov function $V(x, p)$ is decreasing along the trajectory. Therefore, the zero state of system (36) and (39) is Lyapunov stable, and x and $p - \theta$ are bounded. Also note that

$$\begin{aligned}\int_0^T (x^T(t)Qx(t))dt \le V(0, p(0)) - V(x(T), p(T)) \\ \le V(0, p(0)) < \infty\end{aligned} \tag{42}$$

for all $T > 0$, then

$$\int_0^\infty (x^T(t)Qx(t))dt \le V(0, p(0)) < \infty. \tag{43}$$

It can be shown that $x = x(t)$ is uniformly continuous, thus it can be concluded that

$$\lim_{t \to \infty} x(t) = 0, \tag{44}$$

and thereby,

$$\lim_{t \to \infty} E_i(t) = \lim_{t \to \infty} x_1(t) = 0. \tag{45}$$

5. SIMULATION STUDY

In this study, we illustrate the benefit of adaptive control law for a simple formation manipulations of a spacecraft triad. The adaptive control law is used for the formation manipulation and formation keeping. We use the attitude control law from Wang-Hadaegh [4] for the attitude manipulation.

In this example, we choose one spacecraft as the leader, the remaining two spacecraft as followers. The formation manipulation scenario is as follows: all three spacecraft are initially clustered, then they move to a triangular formation (I); after 100 second, the spacecraft move to a new triangular formation (II). Constant disturbance forces

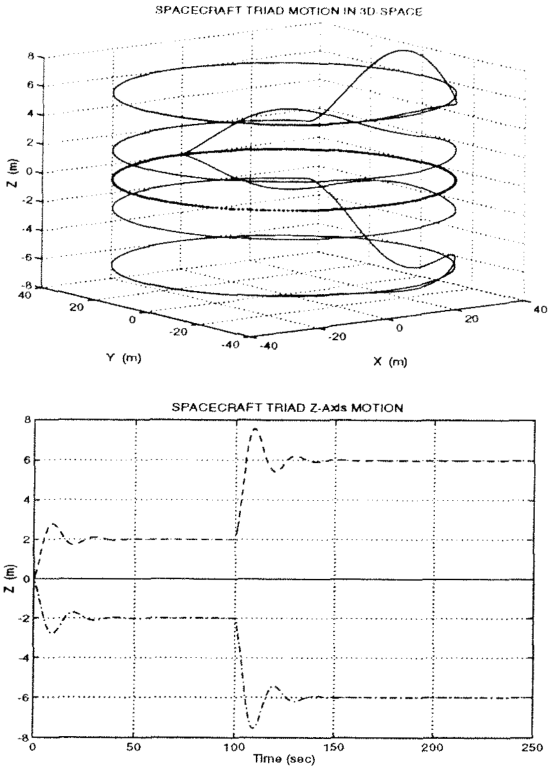

Fig. 2. Upper Plot: Dotted Line: Leader. Lower Plot: Solid Line: Leader; Dashed Lines: Followers

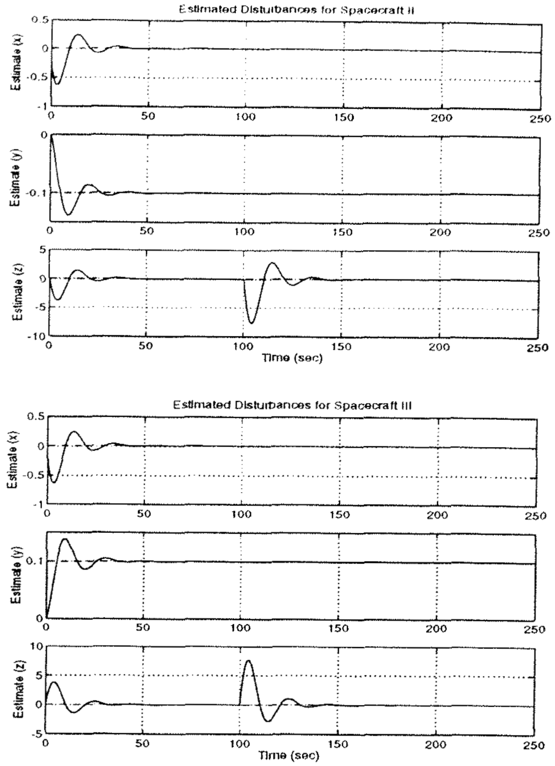

Fig. 3. The Estimated Disturbances on the follower spacecraft

are imposed on the three spacecraft. We assume the disturbance on the leader spacecraft can be measured, and we use the adaptive control laws proposed in this paper for the followers.

The parameters for this simulation are listed in Table I. The simulation results are shown in Figs. 2 and 3.

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Parameter	**Spacecraft I**	**Spacecraft II**	**Spacecraft III**
Mass (kg)	20	10	10
(Radius, Height) (m)	(0.25,0.50)	(0.25,0.50)	(0.25,0.50)
Friction Coefficient (Ns/m)	0.00	0.01	0.01
Disturbance Acceleration (m/s^2)	(0.00, 0.00, 0.00)	(0.01, -0.10, 0.01)	(0.01, 0.10, -0.01)
Feedback Gain Matrix	[1 10]	[1 10]	[1 10]
Formation I ($t \leq 100s$)	(0, 0, 0)	(0, 0, 2)	(0, 0, -2)
Formation II ($t > 100s$)	(0, 0, 0)	(0, 0, 6)	(0, 0, -6)

Table I.